

# SUSY and Precision Observables

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1. Introduction
2. Precision Observables in the MSSM
3. Precision Observables in the CMSSM: collider implications
4. Conclusions

# 1. Introduction

**Q:** Which Lagrangian describes the world?

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**A:** Ummm . . . Let's start differently!

# 1. Introduction

Supersymmetry (SUSY) : Symmetry between

$$\begin{aligned} & \text{Bosons} \leftrightarrow \text{Fermions} \\ Q \text{ |Fermion}\rangle & \rightarrow \text{|Boson}\rangle \\ Q \text{ |Boson}\rangle & \rightarrow \text{|Fermion}\rangle \end{aligned}$$

Simplified examples:

$$\begin{aligned} Q \text{ |top, } t\rangle & \rightarrow \text{|scalar top, } \tilde{t}\rangle \\ Q \text{ |gluon, } g\rangle & \rightarrow \text{|gluino, } \tilde{g}\rangle \end{aligned}$$

$\Rightarrow$  each SM multiplet is enlarged to its double size

**Unbroken SUSY:** All particles in a multiplet have the same mass

Reality:  $m_e \neq m_{\tilde{e}} \Rightarrow$  SUSY is broken ...

... via **soft SUSY-breaking terms** in the Lagrangian (added “by hand”)

**SUSY** particles are made heavy:  $M_{\text{SUSY}} = \mathcal{O}(1 \text{ TeV})$

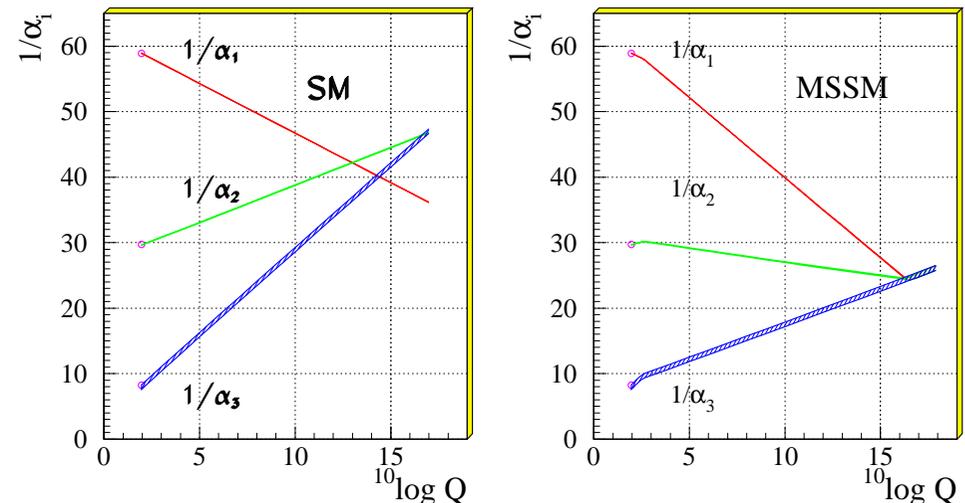
# Supersymmetry: Motivation

The SM is in a pretty good shape.

Why SUSY? (Is it worth to double the particle spectrum?)

- Haag-Lopuszanski-Sohnius theorem
- local SUSY: connection to gravity
- coupling constant unification  $\Rightarrow$
- Higgs mechanism for free  
 $\Rightarrow$  top quark mass prediction
- LSP is good CDM candidate

Unification of the Coupling Constants  
in the SM and the minimal MSSM



$\Rightarrow$  The **Minimal Supersymmetric Standard Model (MSSM)** is a compelling candidate for physics beyond the SM

# The Minimal Supersymmetric Standard Model (MSSM)

## Superpartners for Standard Model particles

$$\begin{array}{llll} [u, d, c, s, t, b]_{L,R} & [e, \mu, \tau]_{L,R} & [\nu_{e,\mu,\tau}]_L & \text{Spin } \frac{1}{2} \\ [\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R} & [\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R} & [\tilde{\nu}_{e,\mu,\tau}]_L & \text{Spin } 0 \\ g & \underbrace{W^\pm, H^\pm}_{\text{Spin } 1} & \underbrace{\gamma, Z, H_1^0, H_2^0}_{\text{Spin } 0} & \text{Spin } 1 / \text{Spin } 0 \\ \tilde{g} & \tilde{\chi}_{1,2}^\pm & \tilde{\chi}_{1,2,3,4}^0 & \text{Spin } \frac{1}{2} \end{array}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

$\tilde{t}$  sector of the MSSM: (scalar partner of the top quark)

Mass matrix for  $\tilde{t}_L, \tilde{t}_R$ :

$$(\tilde{t}_L, \tilde{t}_R) \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + DT_{1t} & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + DT_{2t} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

$\Downarrow$  ← Diagonalization,  $\theta_{\tilde{t}}$

$$(\tilde{t}_1, \tilde{t}_2) \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$

$X_t = A_t - \mu \cot \beta$ ; large mixing possible

⇒ Physical parameters:  $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}$

⇒ Soft SUSY-breaking parameters:  $M_{\tilde{t}_L}, M_{\tilde{t}_R}, A_t$

⇒ Soft SUSY-breaking parameters determine SUSY mass patterns

## Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM  $\Rightarrow m_h^{\text{tree}} \leq M_Z$

physical states:  $h^0, H^0, A^0, H^\pm$

Goldstone bosons:  $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta)$$

## Upper bound on $m_h$ in the MSSM:

“Unconstrained MSSM”:

$M_A$ ,  $\tan \beta$ , 5 parameters in  $\tilde{t}$ - $\tilde{b}$  sector,  $\mu$ ,  $m_{\tilde{g}}$ ,  $M_2$

$$m_h \lesssim 140 \text{ GeV}$$

for  $m_t = 178 \text{ GeV}$

(including theoretical uncertainties from unknown higher orders)

⇒ observable at the LHC

Obtained with:

*FeynHiggs*

[S.H., W. Hollik, G. Weiglein '98, '00, '02]

[T. Hahn, S.H., W. Hollik, G. Weiglein '03, '04]

[www.feynhiggs.de](http://www.feynhiggs.de)

→ all Higgs masses, couplings, BRs (easy to link, easy to use :-)

**A:** Let's try again . . .

**Q:** Which Lagrangian describes the world?

**Q':** What describes the world better: SM or MSSM ?

**A:** Two possible ways:

- Search for new SUSY particles

new SUSY particles found



SM ruled out

Problem:

SUSY particles are too heavy for today's colliders, only lower limits of  $\mathcal{O}(100 \text{ GeV})$ .

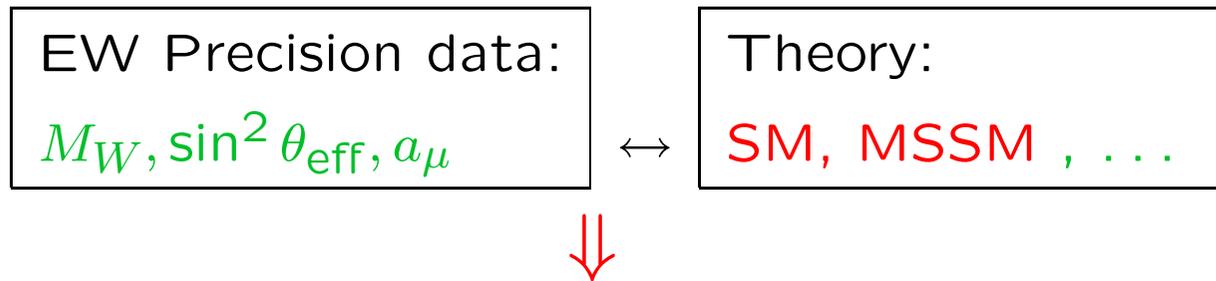
→ waiting for Tevatron (2006/07...?)

→ waiting for LHC (2008/09...?)

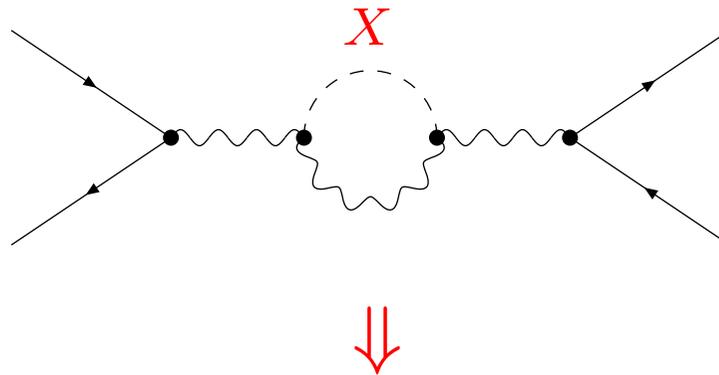
- Search for indirect effects of SUSY  
via Precision Observables

## Precision Observables (POs):

Comparison of electro-weak precision observables with theory:



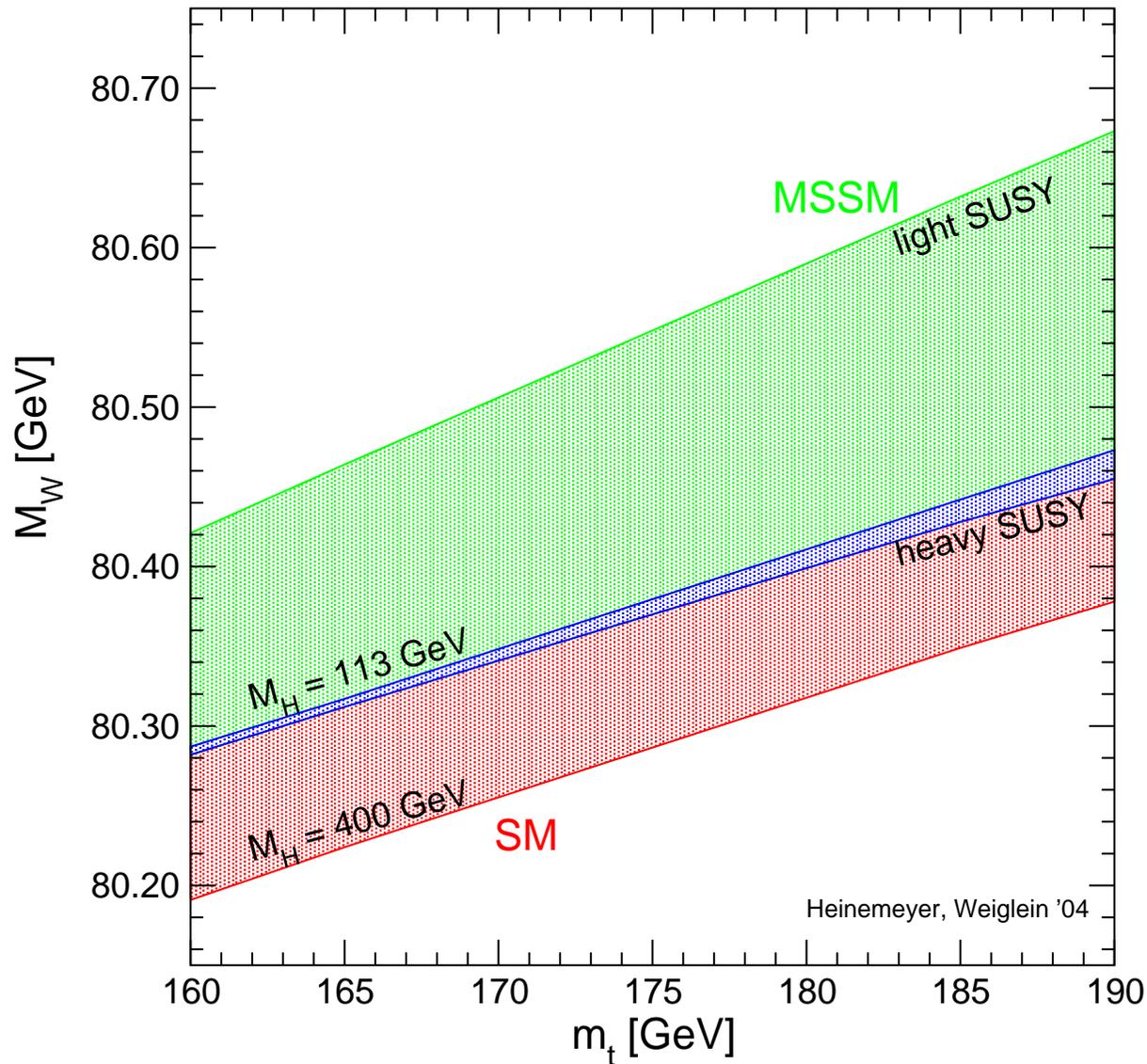
Test of theory at quantum level: Sensitivity to loop corrections



Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

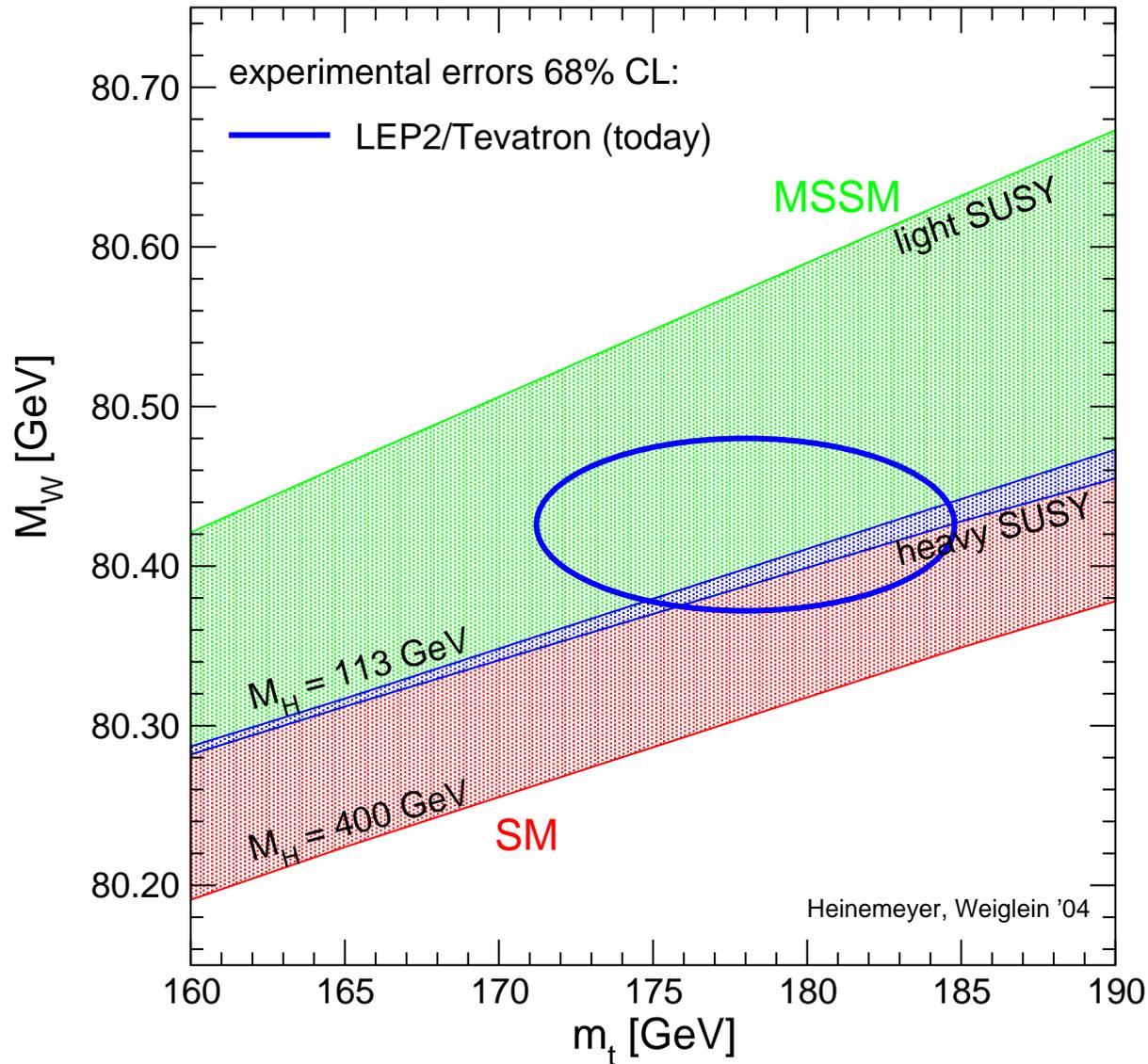
Example: Prediction for  $M_W$  in the SM and the MSSM :



MSSM uncertainty:  
unknown masses  
of SUSY particles

SM uncertainty:  
unknown Higgs mass

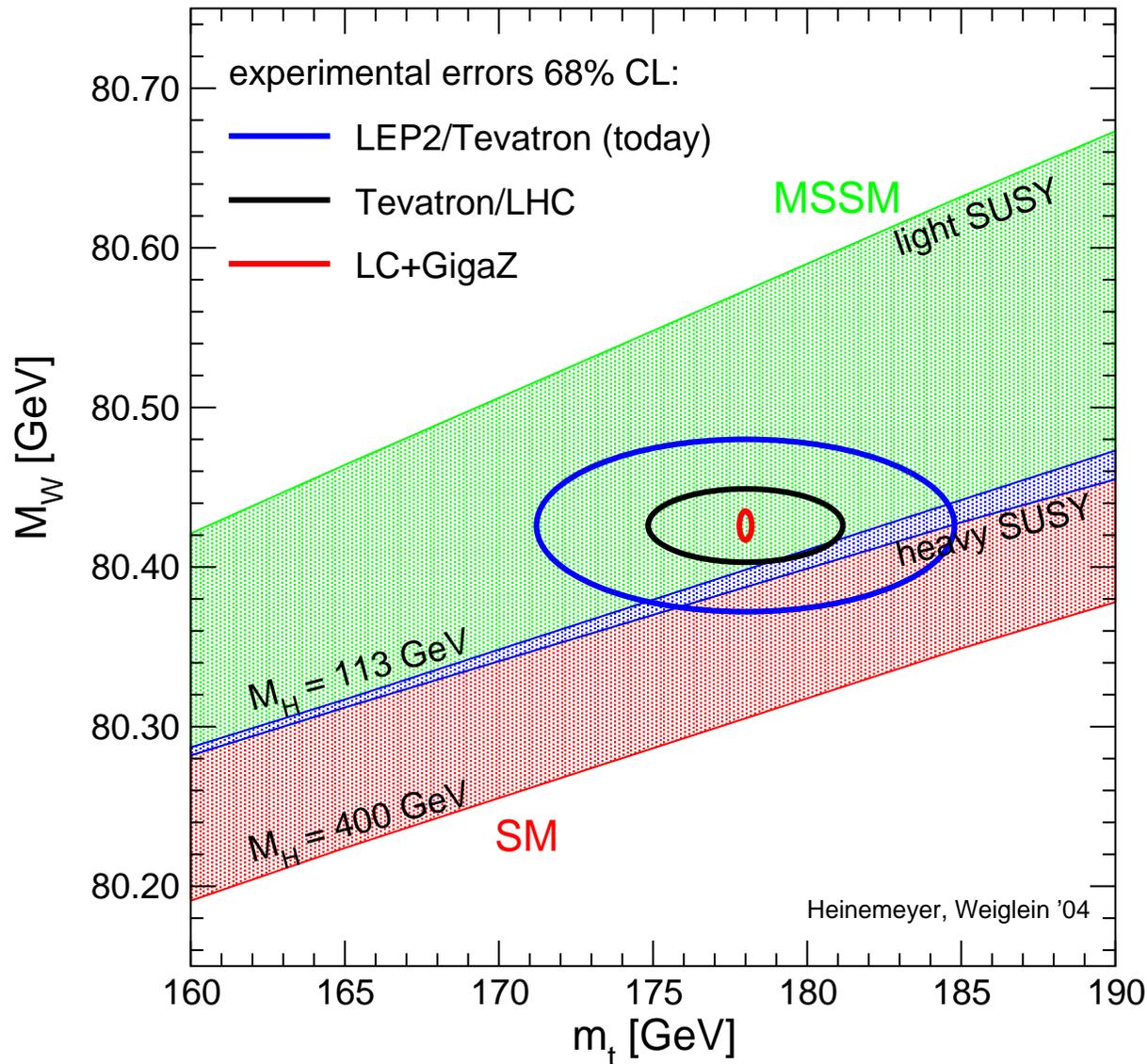
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## 2. Precision Observables in the MSSM

Precision observables:  $M_W$ ,  $\sin^2 \theta_{\text{eff}}$ ,  $m_h$ ,  $(g-2)_\mu$ ,  $b$  physics, ...

- 1.) Theoretical prediction for  $M_W$  in terms of  $M_Z$ ,  $\alpha$ ,  $G_\mu$ ,  $\Delta r$ :

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} \left( \frac{1}{1 - \Delta r} \right)$$



loop corrections

- 2.) Effective mixing angle:

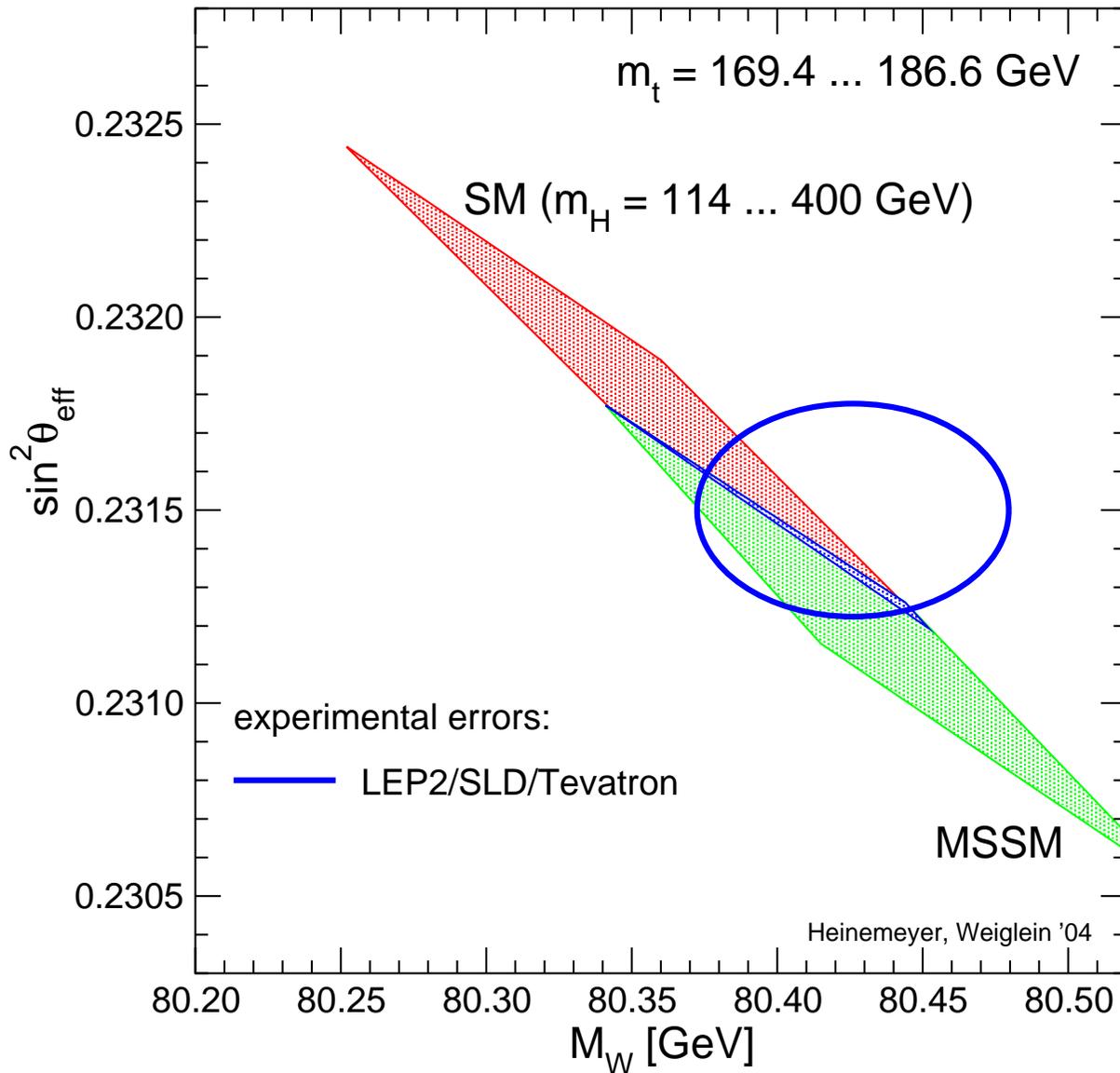
$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left( 1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

## Example of application:

Prediction for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  in the SM and the MSSM :

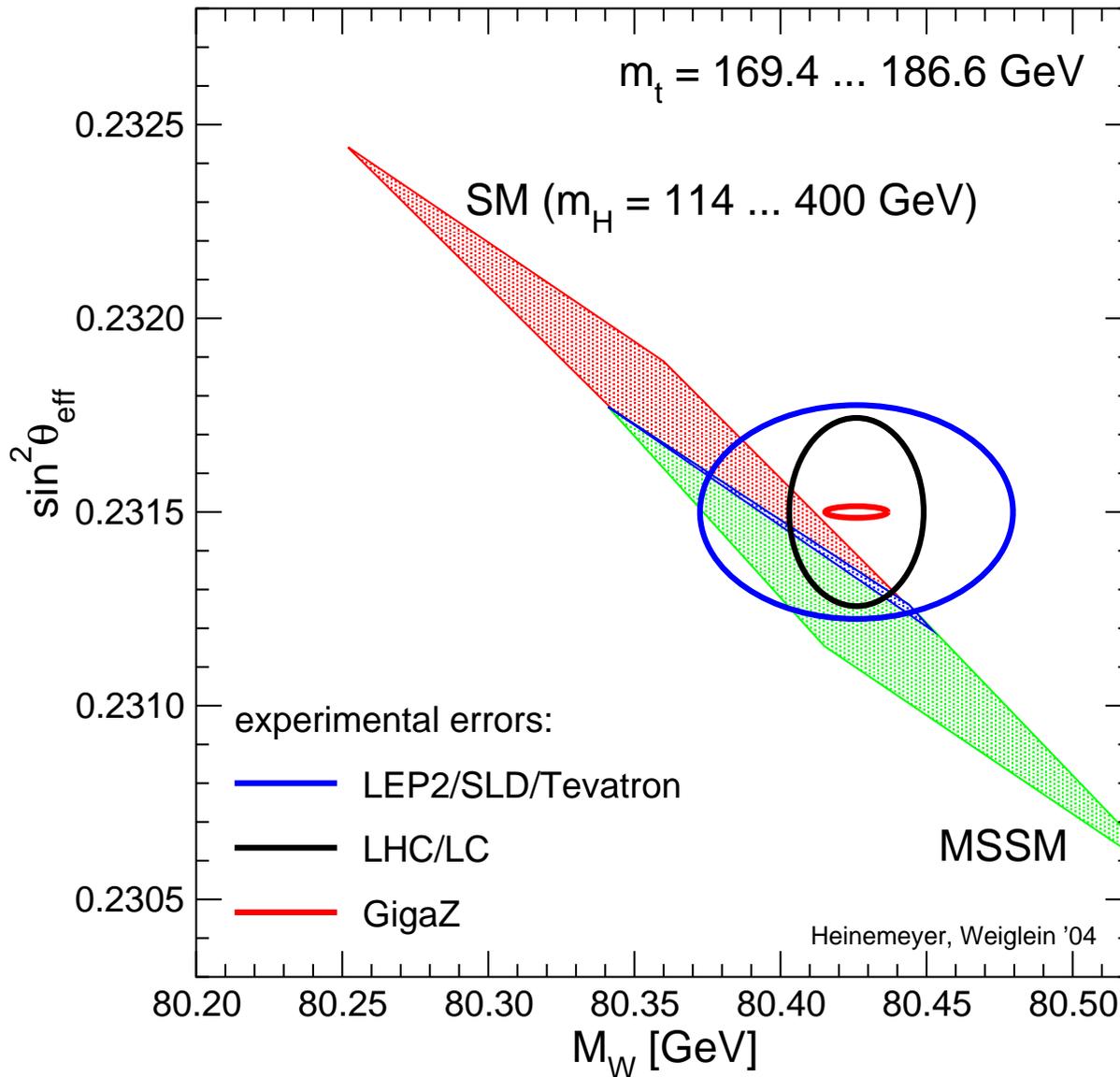


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MSSM uncertainty:  
unknown masses  
of SUSY particles

SM uncertainty:  
unknown Higgs mass

3.) Theoretical prediction of the lightest MSSM Higgs boson mass:  $m_h$

Contrary to the SM:  $m_h$  is not a free parameter

MSSM tree-level bound:  $m_h < M_Z$ , excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:  $\Delta m_h^2 \sim G_\mu m_t^4 \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

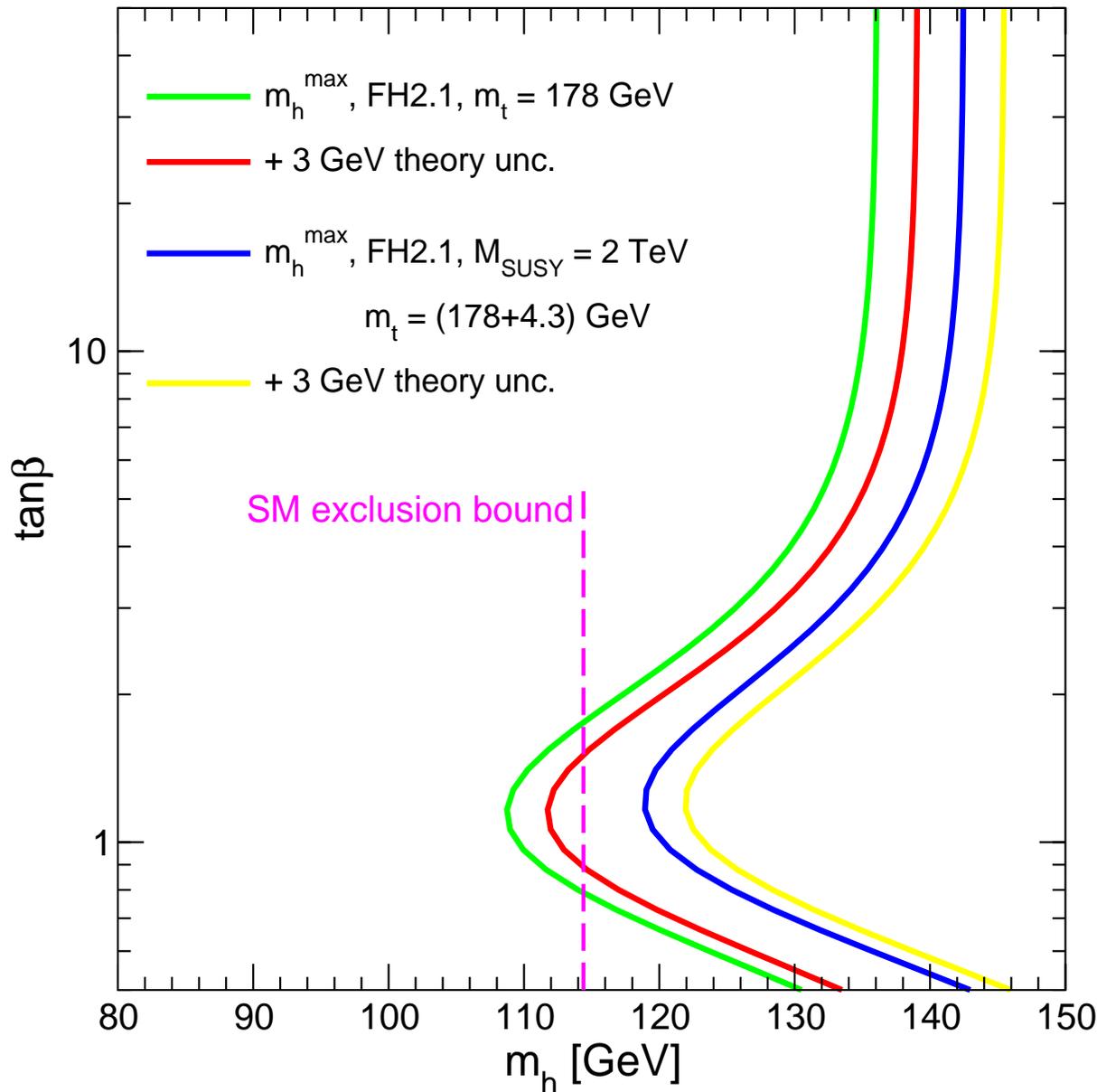
Measurement of  $m_h$ , Higgs couplings  $\Rightarrow$  test of the theory

LHC:  $\Delta m_h \approx 0.2$  GeV

ILC:  $\Delta m_h \approx 0.05$  GeV

$\Rightarrow m_h$  will be (the best?) electroweak precision observable

## Example of application: effect on $\tan\beta$ exclusion bounds



compare:

$m_h^{\max}$  scenario, *FeynHiggs2.1*,  
 $m_t = 178$  GeV

$m_h^{\max}$  scenario, *FeynHiggs2.1*,  
 3 GeV theory unc.

$\Rightarrow \tan\beta$  bound considerably  
 weakened

even worse:

$m_h^{\max}$  scenario with  
 $m_t = 182.3$ ,  $M_{\text{SUSY}} = 2$  TeV

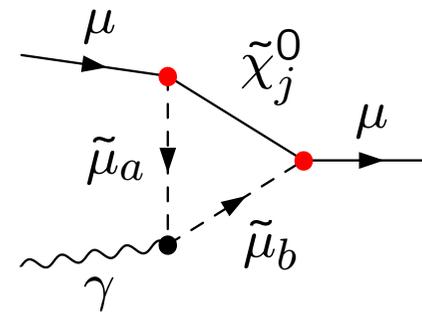
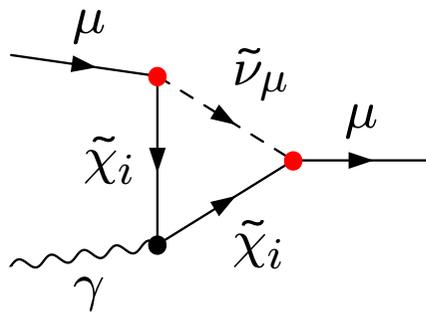
$m_h^{\max}$  scenario with  
 $m_t = 182.3$ ,  $M_{\text{SUSY}} = 2$  TeV  
 + 3 GeV theory unc.

4.) Prediction of the **anomalous magnetic moment of the muon**:  $(g - 2)_\mu$

Coupling of **muon** to **magnetic field** :  $\mu - \mu - \gamma$  coupling

$$\bar{u}(p') \left[ \gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = (g - 2)_\mu$$

Feynman diagrams for MSSM 1L corrections:



Enhancement factor as compared to SM:

$$\begin{aligned} \mu - \tilde{\chi}_i^\pm - \tilde{\nu}_\mu & : \sim m_\mu \tan \beta \\ \mu - \tilde{\chi}_j^0 - \tilde{\mu}_a & : \sim m_\mu \tan \beta \end{aligned}$$

## Overview of the SM theory evaluation:

Source	contr. to $a_\mu [10^{-10}]$	
LO hadr.	$\sim 695 \pm 7 (e^+e^-)$	[Davier et al, Hagiwara et al. '03]
		[Ghozzi, Jegerlehner '03]
	$711.0 \pm 6 (\tau)$	[Davier, Eidelman, Höcker, Zhang '03]
LBL	$8 \pm 4$	[Knecht, Nyffeler '02]
	$13.6 \pm 2.5$ tbc	[Melnikov, Vainshtein '03]
EW 1L	19	
EW 2L	-4	[Czarnecki, Krause, Marciano '98]
exp. res.	6	[BNL E821 '04]

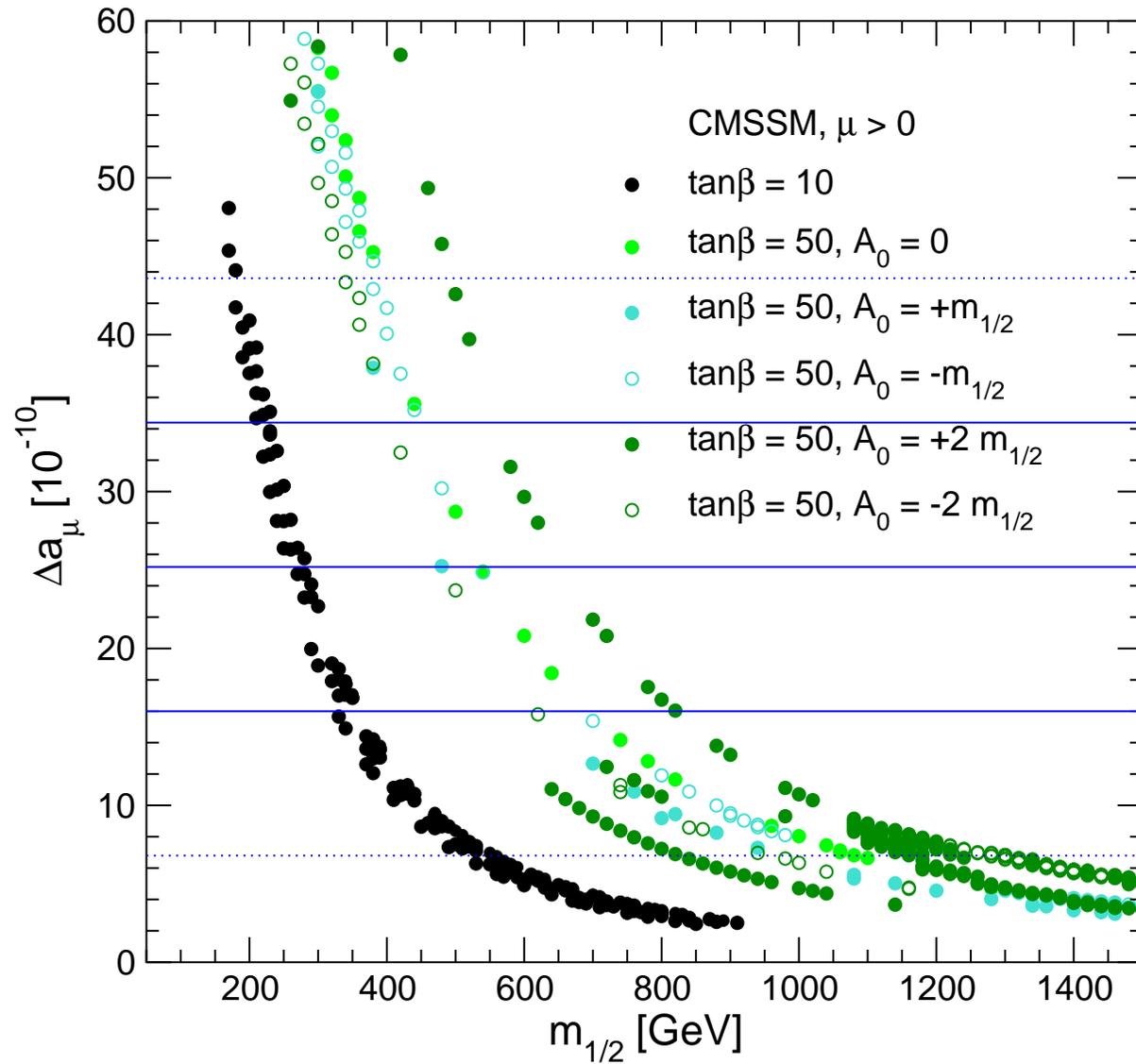
→ “Isospin breaking effects” in  $\tau$  data problematic [Ghozzi, Jegerlehner '03]

→ KLOE data (radiative return) agrees with  $e^+e^-$  data

⇒ general agreement at ICHEP'04 Beijing: discard  $\tau$  data

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (25.2 \pm 9.2) \times 10^{-10}$$

## Example: Investigation of mSUGRA with cold dark matter constraint



Scan over  $m_{1/2}, m_0, A_0$   
 $\tan\beta = 10, 50$   
selected points give correct  
amount of cold dark matter

[Ellis, S.H., Olive, Weiglein '04]

Severe bounds on e.g.  $m_{1/2}$

These are the current prediction. But what about the **ERRORS ?**

Three different types of errors:

Experimental error ( $\Rightarrow$  included in the figure):

- current error
  - future expectations
- $\Rightarrow$  sets the scale, has to be matched by other errors

Theory error:

- $\Rightarrow$  error due to missing higher order corrections
- only estimates possible
  - even more complicated for the future

Parametric error:

- current uncertainty in the prediction due to error in the input parameters
  - future uncertainty
- $\Rightarrow$  focus on SM parameters
- e.g.  $\delta m_t^{\text{exp}} \approx \delta m_h^{\text{para}}$
- $\Rightarrow$  derive information about (unknown) SUSY parameters  
(SUSY parametric uncertainties highly model dependent)

### 3. Precision Observables in the CMSSM: collider implications

What is the “CMSSM” or “mSUGRA” ?

⇒ Scenario characterized by

$$m_0, m_{1/2}, A_0, \tan \beta, \text{sign } \mu$$

$m_0$  : universal scalar mass parameter

$m_{1/2}$  : universal gaugino mass parameter

$A_0$  : universal trilinear coupling

$\tan \beta$  : ratio of Higgs vacuum expectation values

$\text{sign}(\mu)$  : sign of supersymmetric Higgs parameter

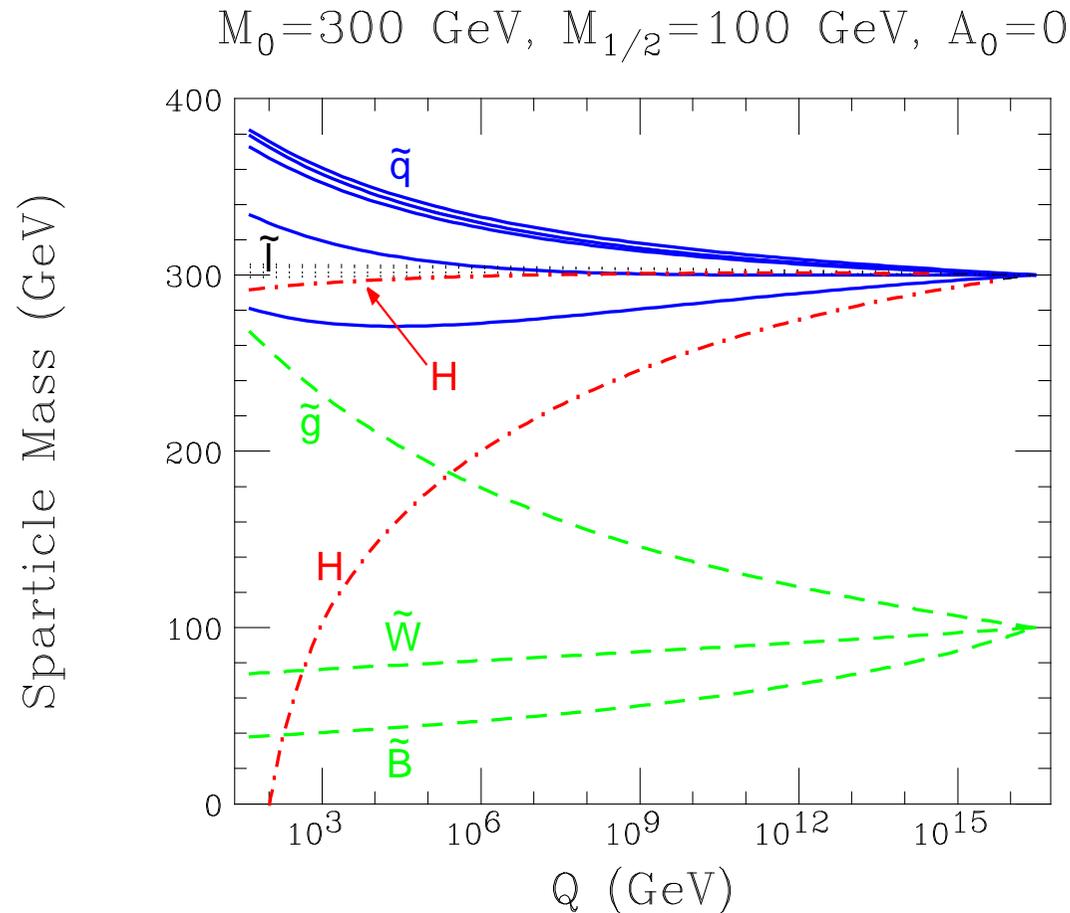
} at the GUT scale

⇒ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is usually lightest neutralino

Low-energy parameters (at the electroweak (EW) scale) via  
 "Renormalization group equations" (RGEs)

[RGE: equations that connect parameters at different energy scales]



Note: one parameter in the Higgs potential becomes negative  
 $\Rightarrow$  Higgs mechanism for free

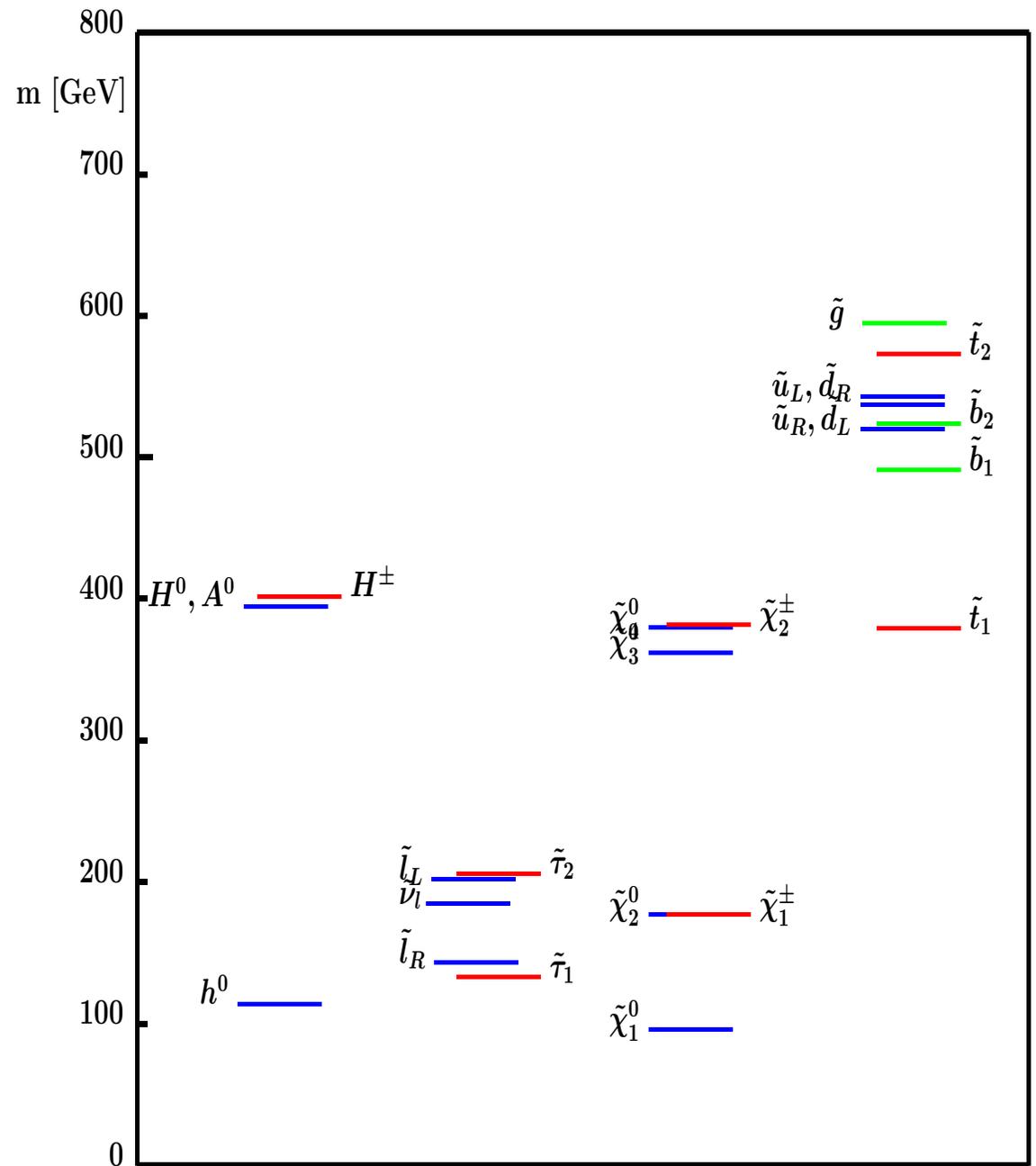
“Typical” CMSSM scenario  
 (SPS 1a benchmark scenario):

SPS home page:

[www.ippp.dur.ac.uk/~georg/sps](http://www.ippp.dur.ac.uk/~georg/sps)

$\Rightarrow m_h \lesssim 130 \text{ GeV}$

$\Rightarrow$  observable at the Tevatron



## Precision Observables in the CMSSM: collider implications

What do we know about the SUSY mass scale?

1. Coupling constant unification  $\Rightarrow M_{\text{SUSY}} \approx 1 \text{ TeV}$
2. LSP should be cold dark matter  $\Rightarrow M_{\text{SUSY}} \lesssim 1 \text{ TeV}$
3. Indirect hints from existing data?

[J. Ellis, S.H., K. Olive, G. Weiglein '04]

– Focus on **mSUGRA/CMSSM**

free parameters:  $m_{1/2}, m_0, A_0, \tan \beta$

– hard constraint: **LSP** gives right amount of **cold dark matter**

only thin **strips** allowed in the  $m_{1/2}$ - $m_0$  plane

fix  **$\tan \beta = 10, 50$**  (lower/upper edge in CMSSM) and  $\mu > 0$

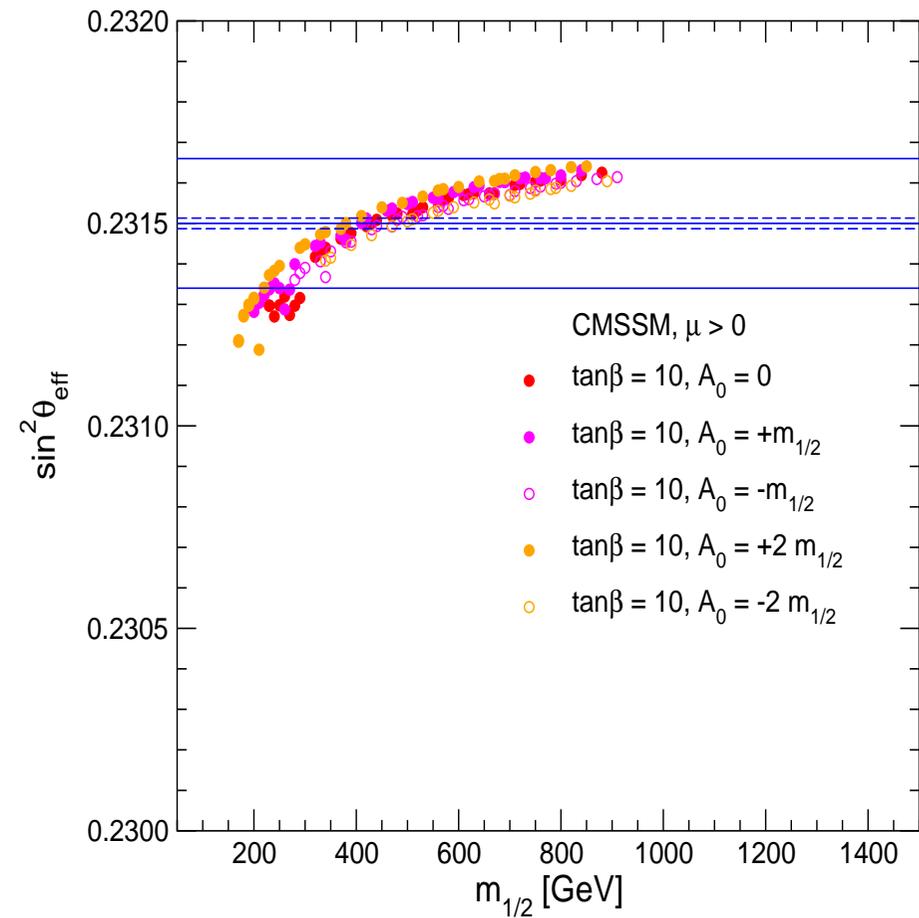
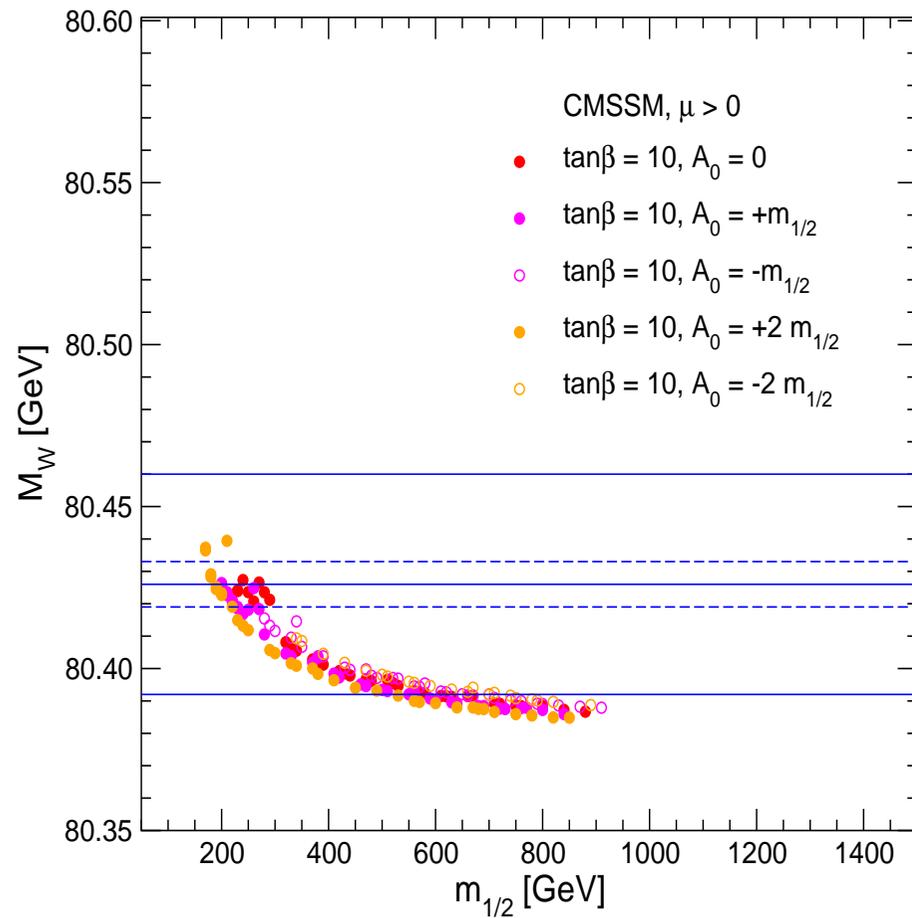
– Use existing data of  $M_W, \sin^2 \theta_{\text{eff}}, \text{BR}(b \rightarrow s\gamma), (g-2)_\mu$

$\Rightarrow \chi^2$  fit with these observables

determine **best fit values** of  $m_{1/2}, m_0, A_0$

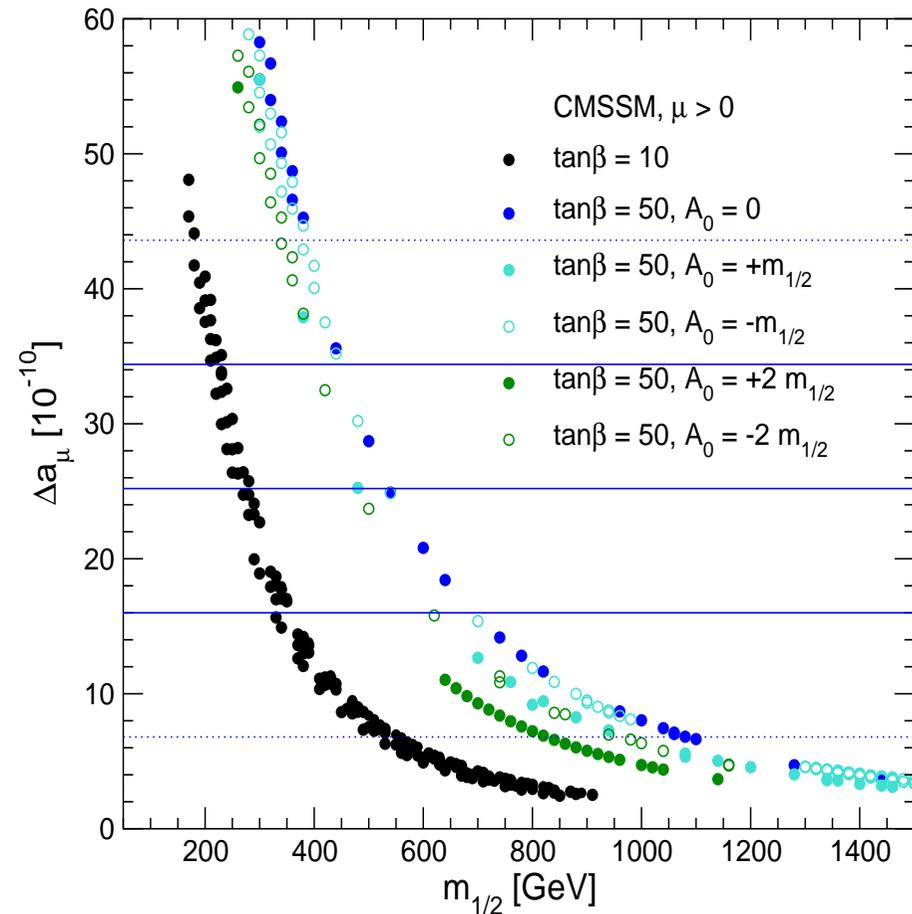
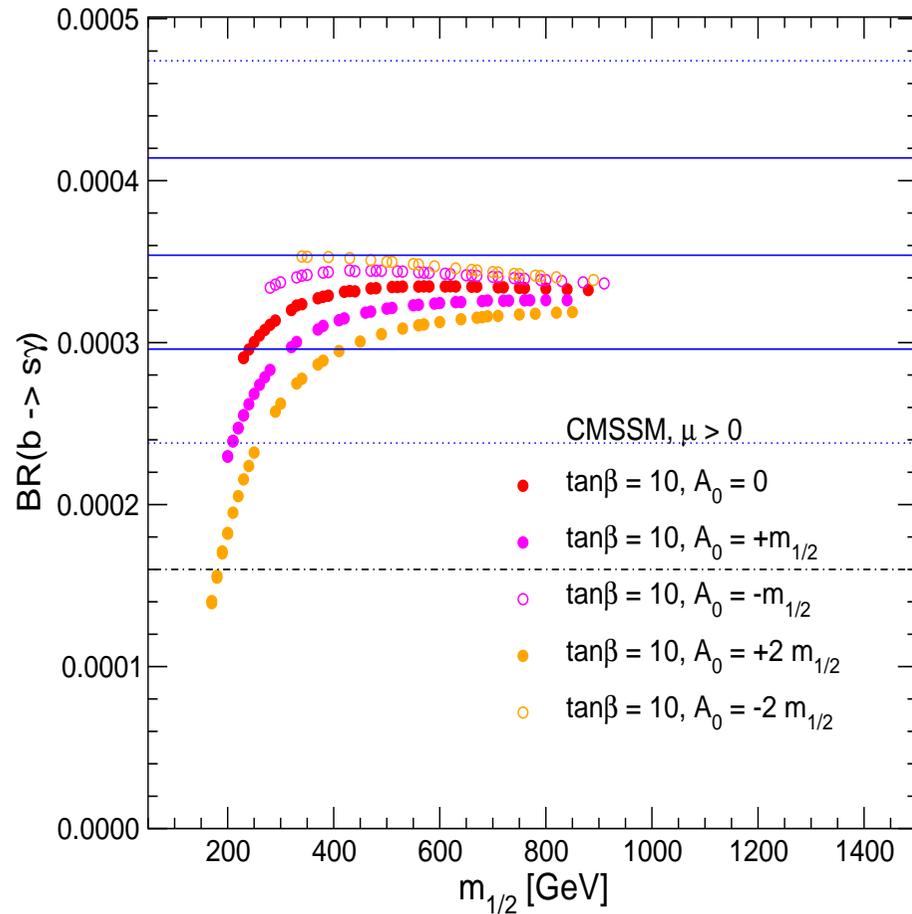
$\Rightarrow$  **best fit values** for masses, couplings, ...

## $M_W$ and $\sin^2 \theta_{\text{eff}}$ for $\tan \beta = 10$ :



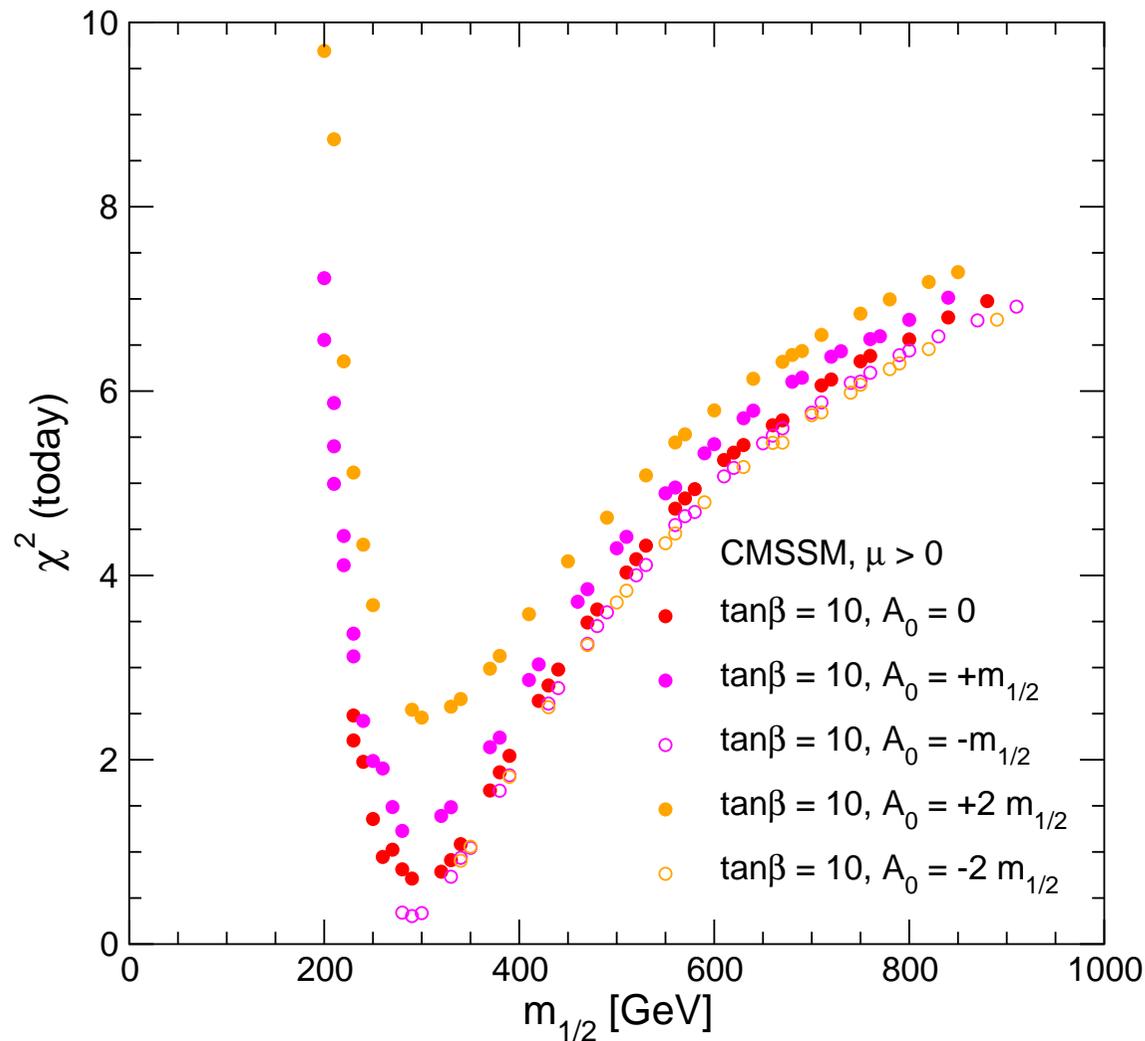
$\Rightarrow m_{1/2} \approx 300$  GeV favored

## BR( $b \rightarrow s\gamma$ ) and $(g - 2)_\mu$ (for $\tan\beta = 10$ ):



$\Rightarrow m_{1/2} \approx 300$  GeV favored

$\chi^2$  fit result for  $m_{1/2}$ : ( $\tan\beta = 10$ ,  $A_0/m_{1/2}$  varied)



Very good fit!

Best fit obtained for

$m_{1/2} \approx 300$  GeV

$A_0 \approx -300$  GeV

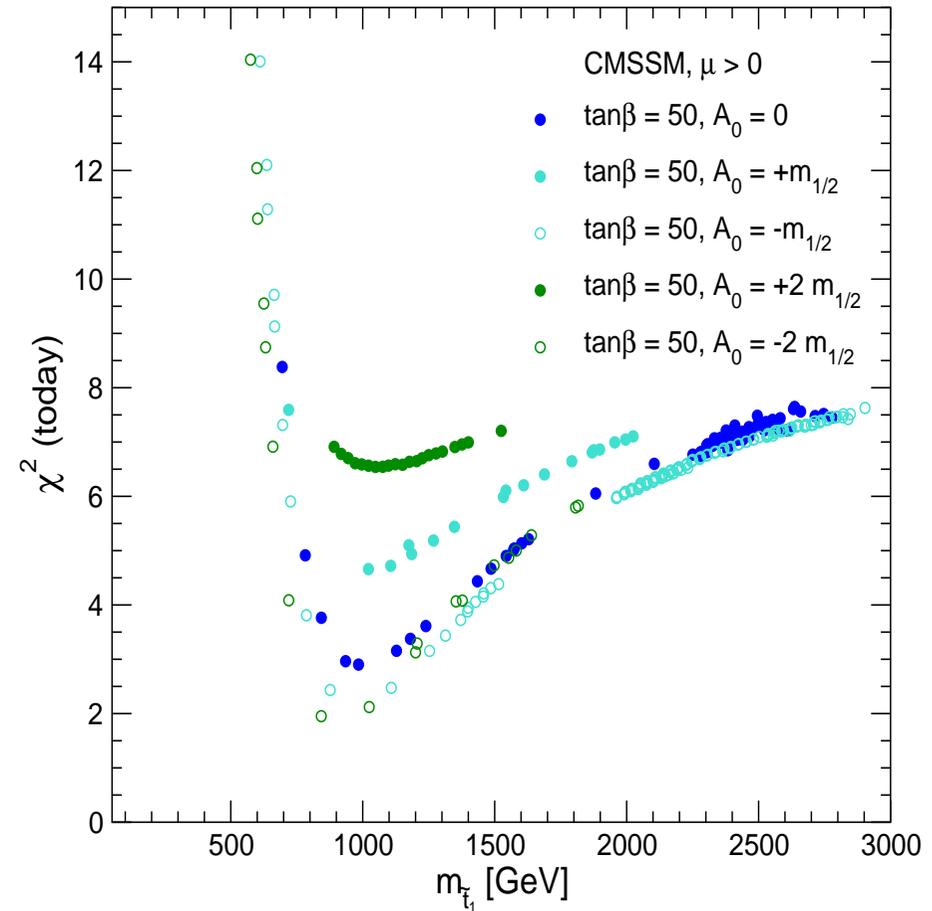
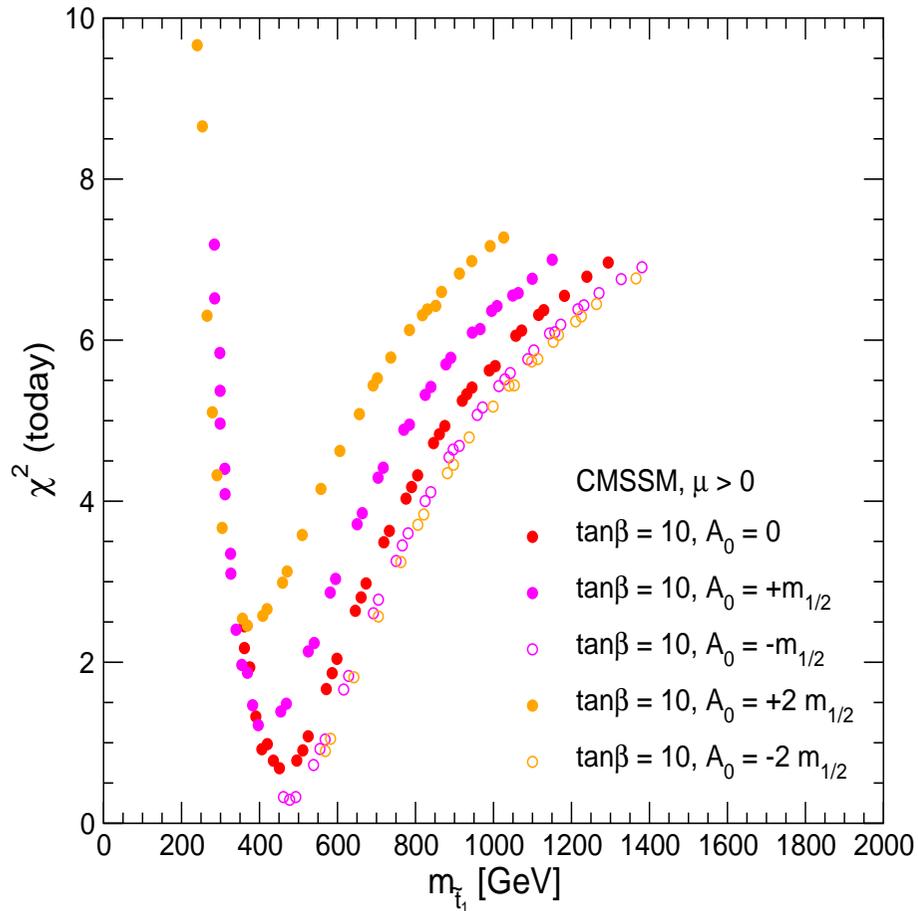
$m_0 \approx 60$  GeV

⇒ SUSY particles relatively light

⇒ very good prospects for the LHC/ILC

slightly worse for  $\tan\beta = 50$

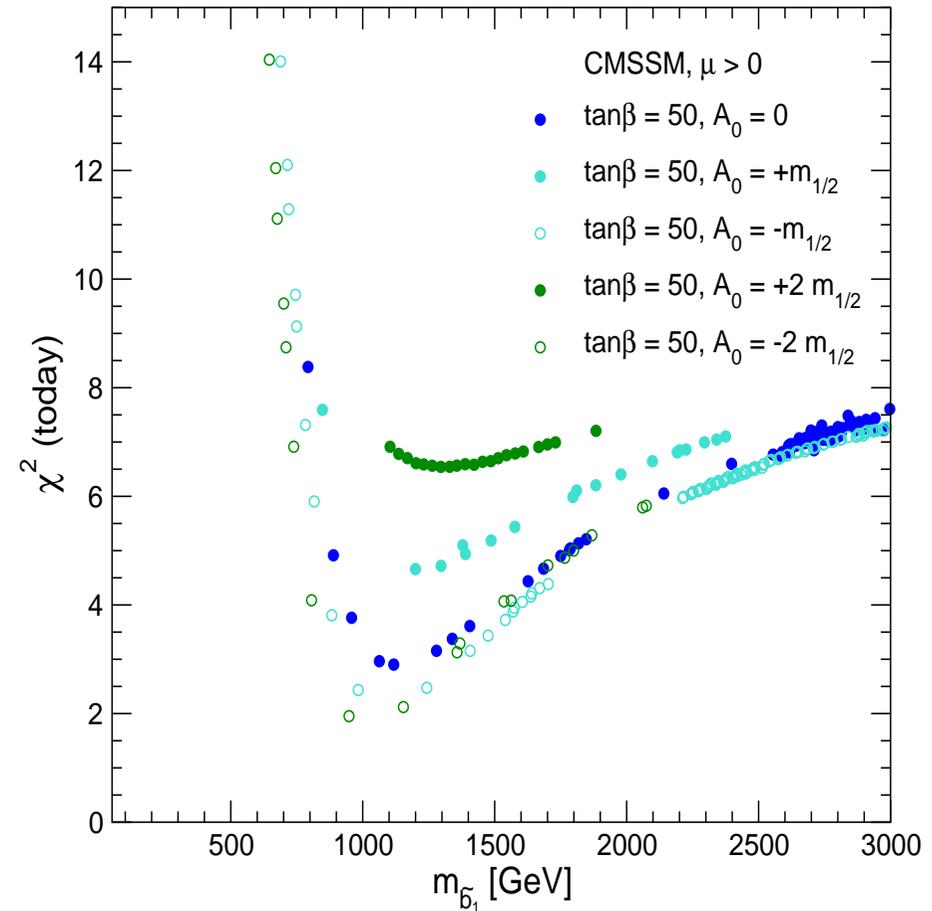
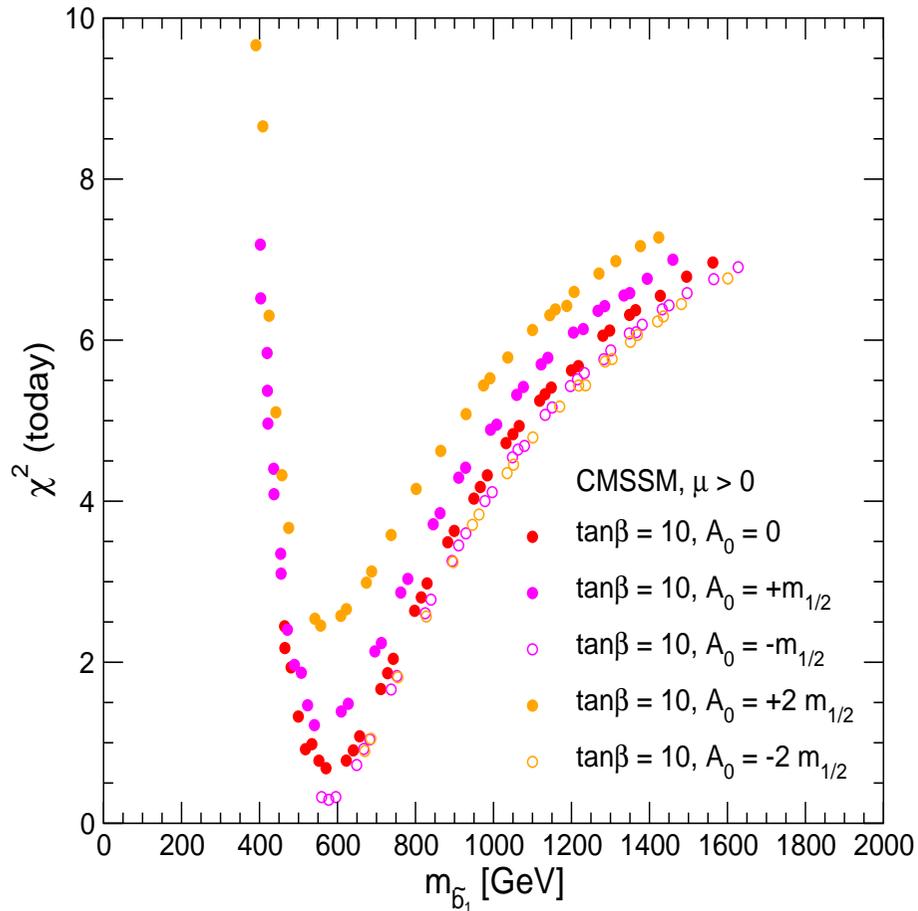
## lightest stop mass for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$  very good prospects for the LHC, not too good for the ILC

$\tan\beta = 50 \Rightarrow$  still quite good for the LHC

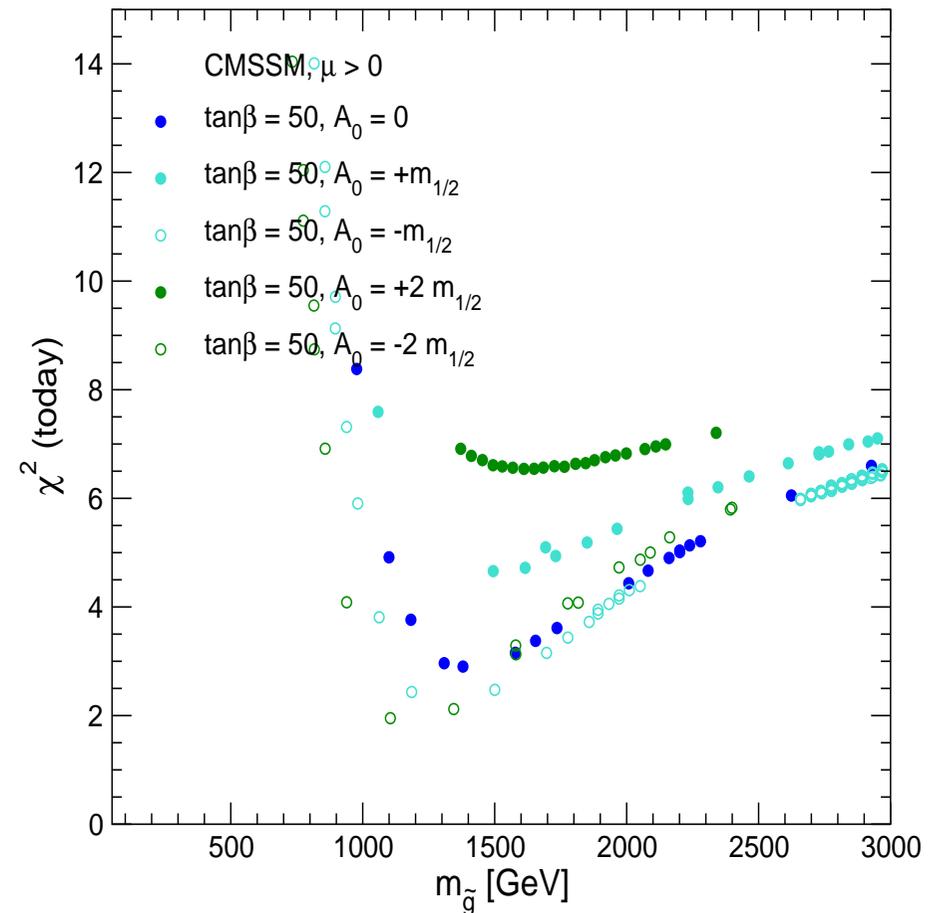
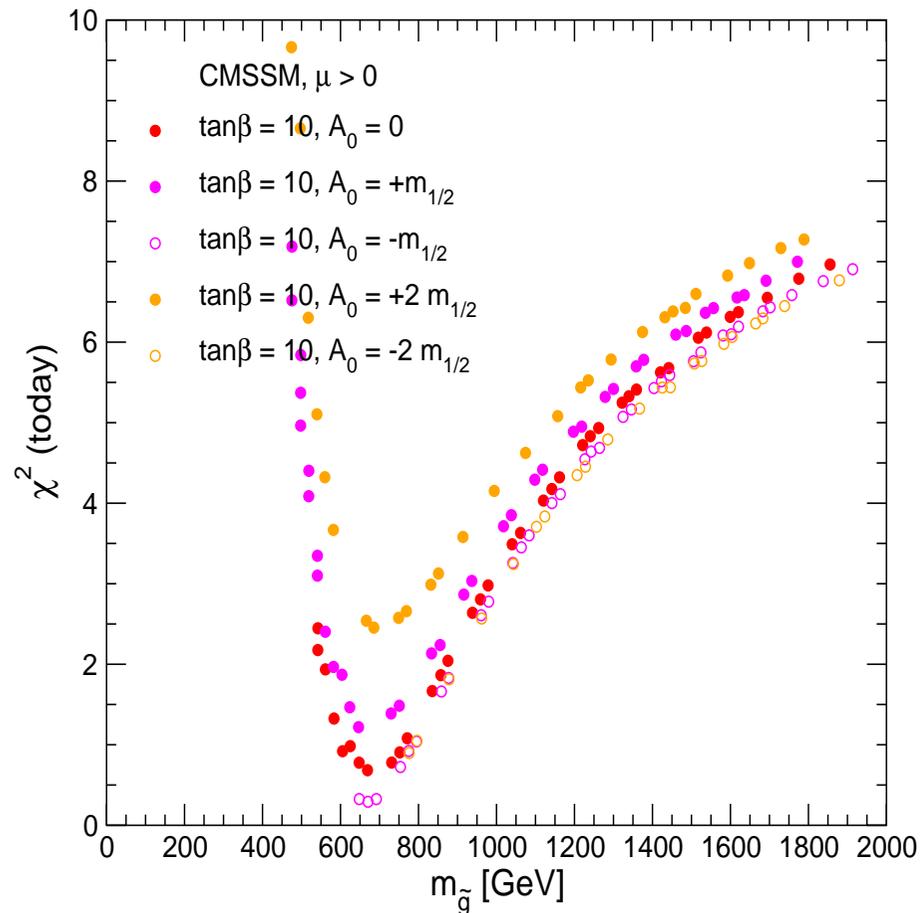
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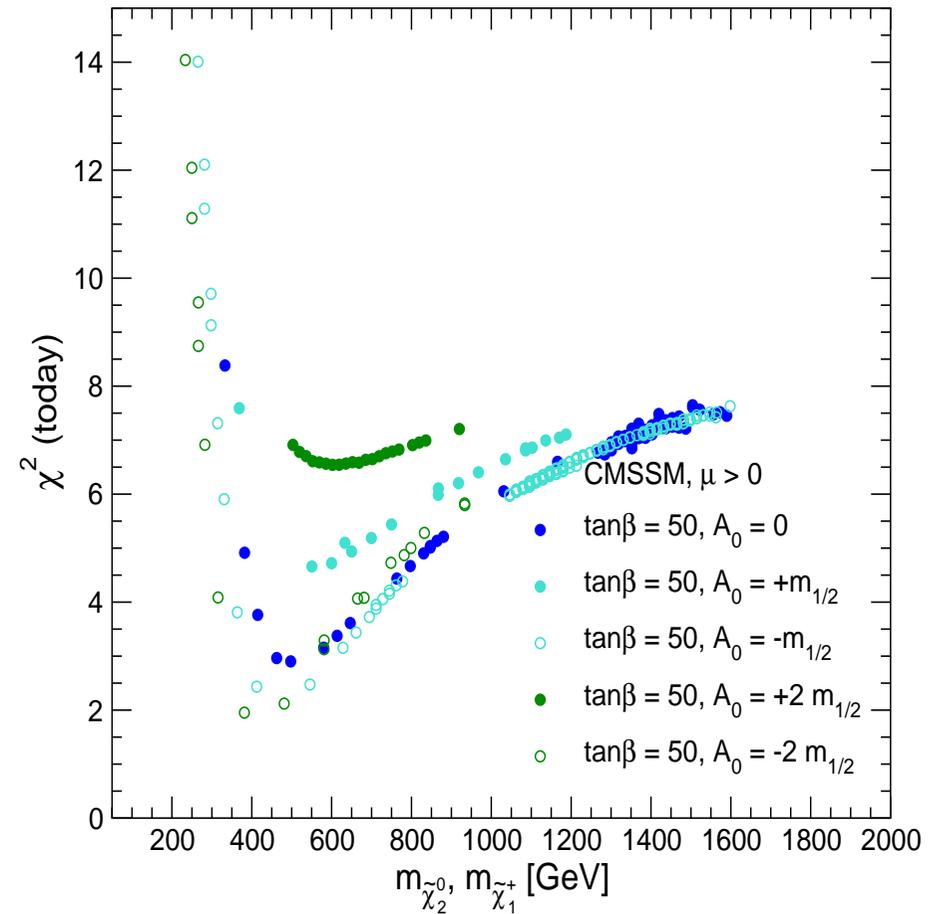
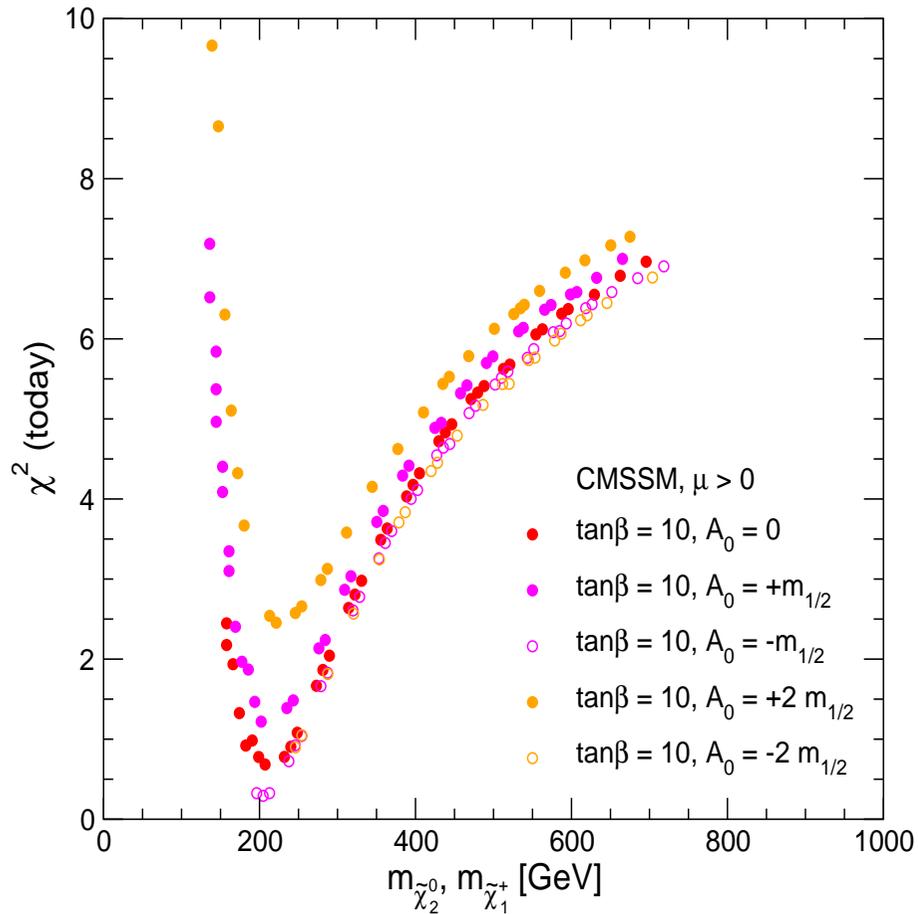
## gluino mass for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$  very good prospects for the LHC, hopeless for the ILC

$\tan\beta = 50 \Rightarrow$  still quite good for the LHC

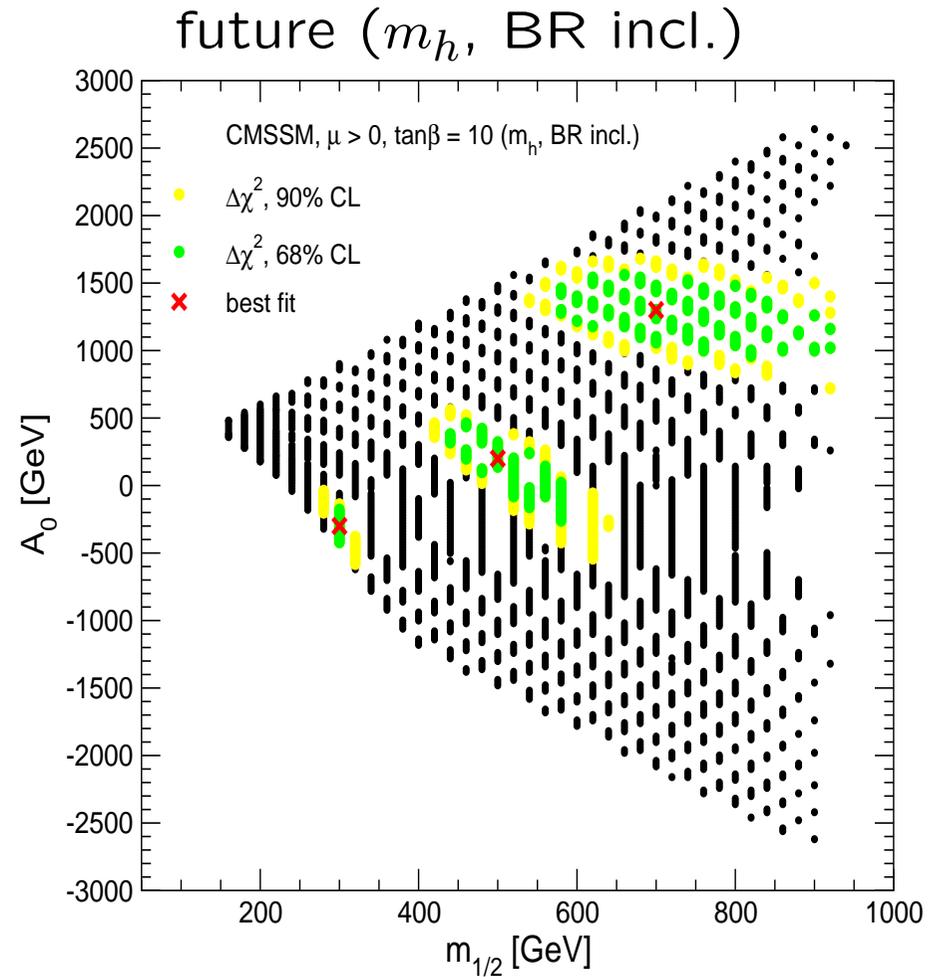
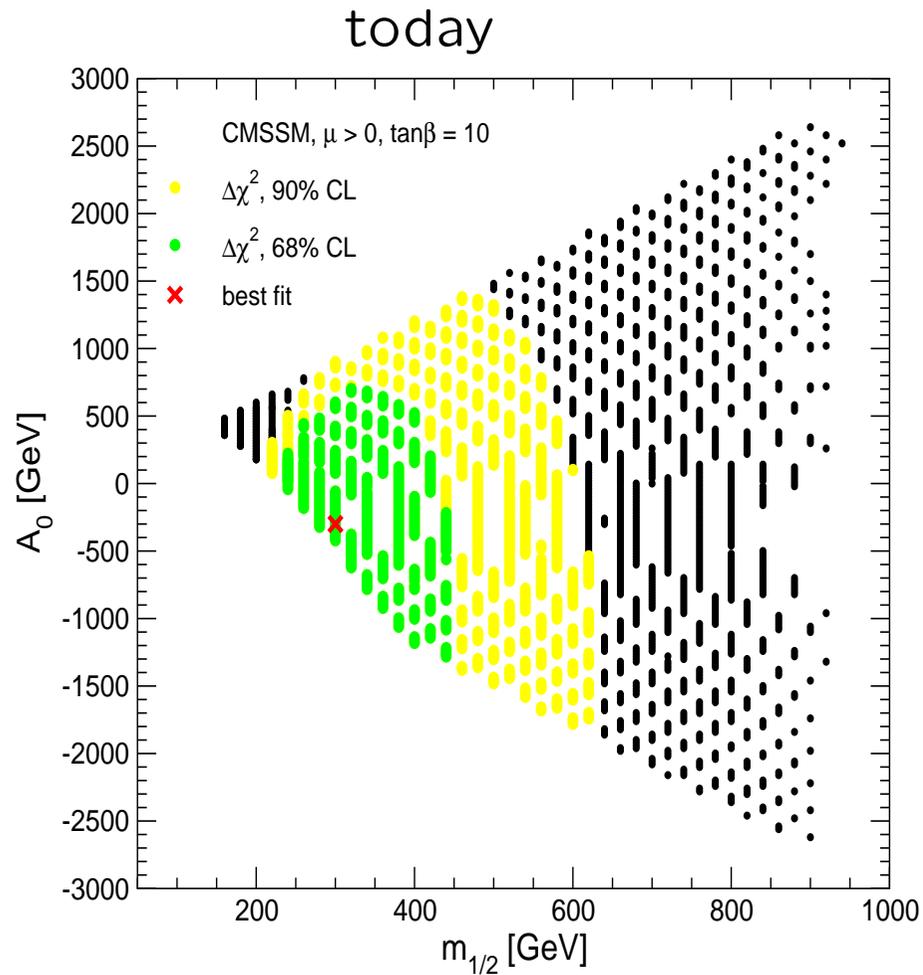
# lightest chargino/next-to-lightest neutralino mass for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$  very good prospects for both, LHC and ILC

$\tan\beta = 50 \Rightarrow$  still quite good

## Compare current and future fit:



Future reach of precision observables larger than direct collider reach

## 4. Conclusinos

- Precision observables
  - can give valuable information about the “true” Lagrangian
  - can provide bounds on SUSY parameter space
- Most important electroweak precision observables:  
 $M_W, \sin^2 \theta_{\text{eff}}, m_h, (g - 2)_\mu, b$  physics
- Three types of errors:
  - experimental: sets the scale
  - intrinsic: unknown higher-order corrections (what we are working on)
  - parametric: exp. error on input parameters (experimentalists work)
- Current  $\chi^2$  fit: indication for not too large SUSY masses
  - ⇒ good prospects for the LHC and the ILC
  - ⇒ future indirect reach larger than kinematical collider limit

## Experimental situation:

### Current/future Experiments

→ provide high accuracy **measurements** !

## Theory situation:

measured **observables** have to be compared with **theoretical predictions**  
(of your favorite model)

**Measured data** is only meaningful if it is matched with  
**theoretical calculations** at the **same level of accuracy**

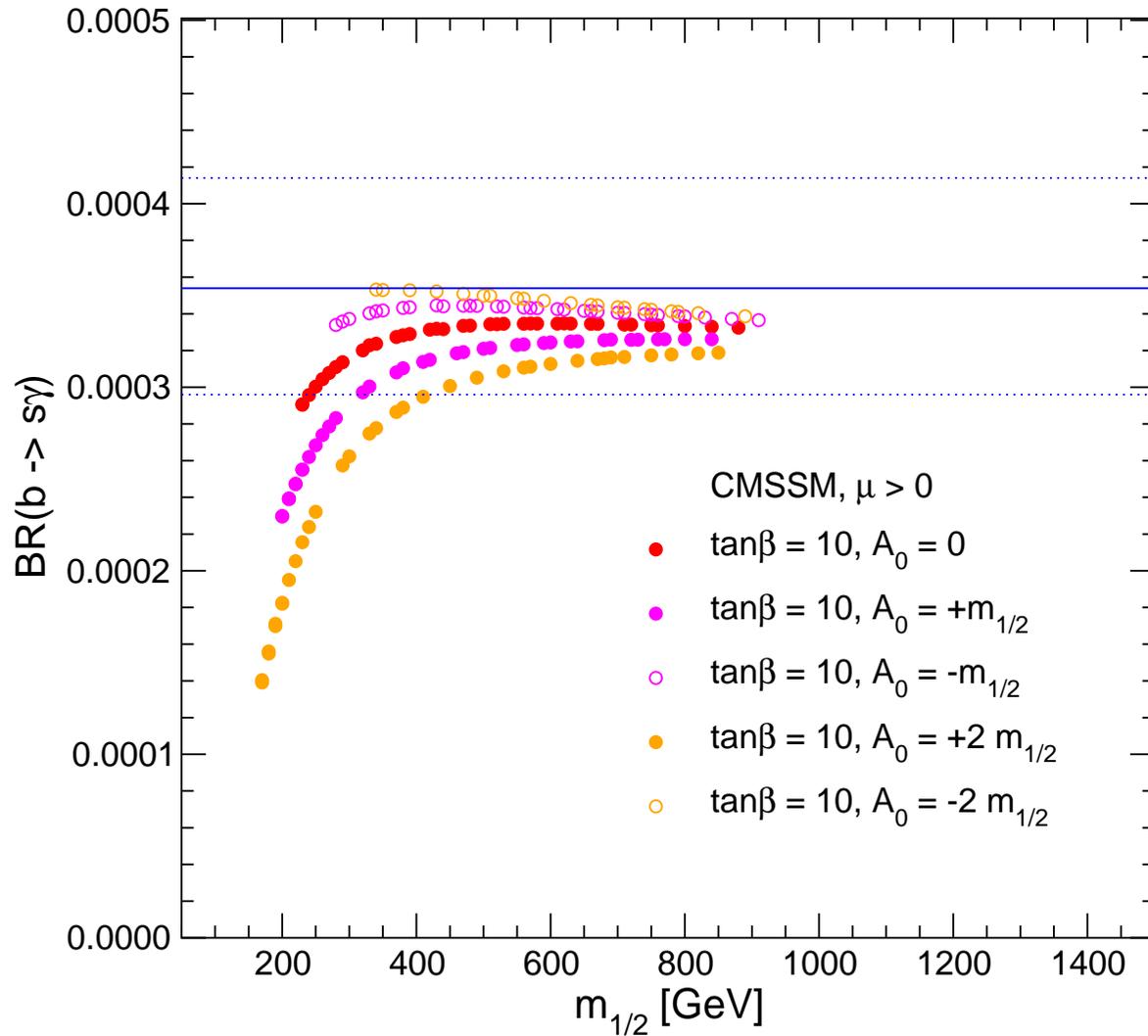
We have to start **NOW** to achieve necessary accuracy in time

Theoretical calculations should be viewed as an essential part of all  
future High Energy Physics programs

# Backup slides

# Investigation of mSUGRA with cold dark matter constraint:

## BR( $b \rightarrow s\gamma$ ), $\tan\beta = 10$



Scan over  $m_{1/2}, m_0, A_0$

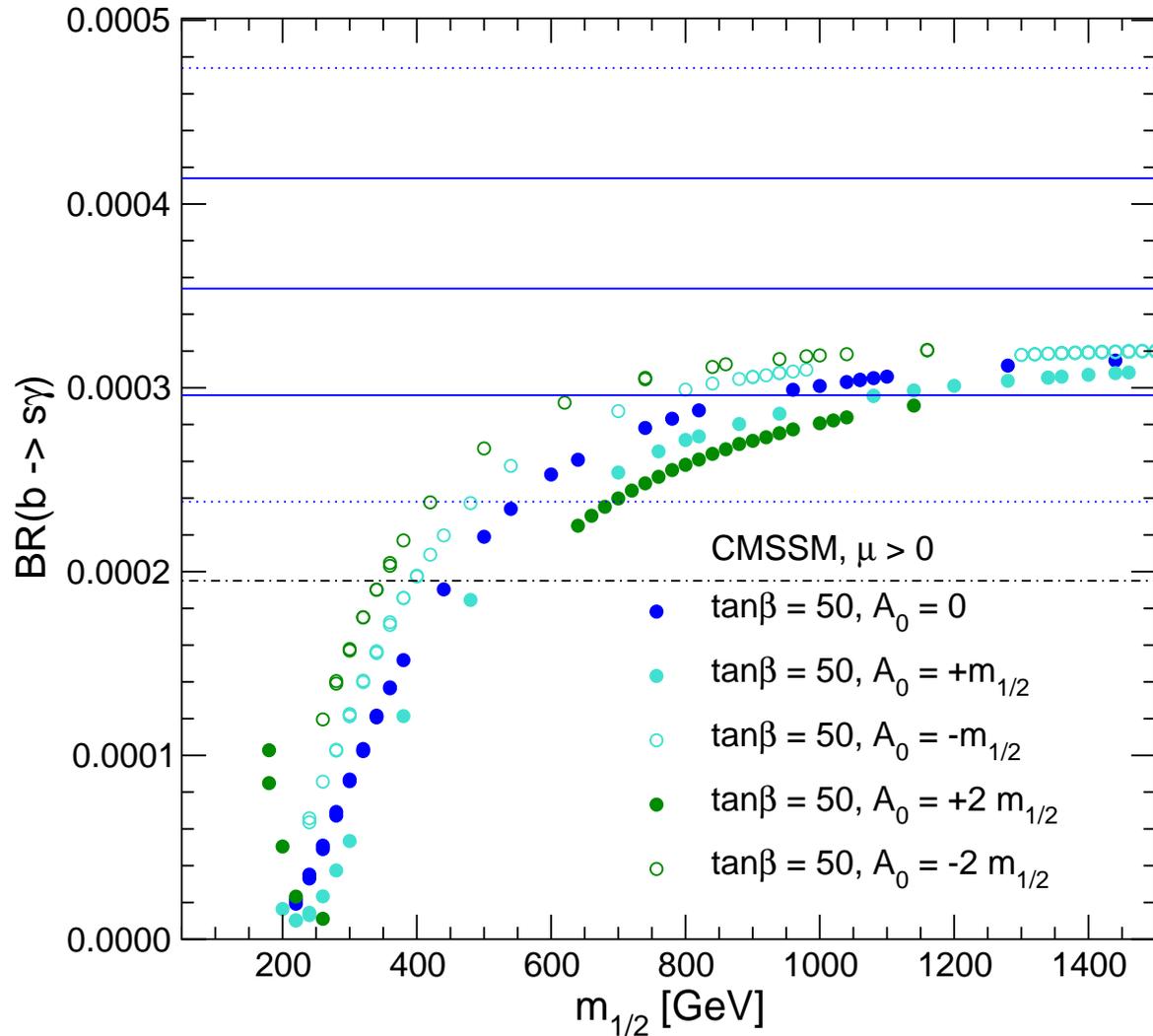
$\tan\beta = 10$

selected points give correct  
amount of cold dark matter

[Ellis, S.H., Olive, Weiglein '04]

# Investigation of mSUGRA with cold dark matter constraint:

## BR( $b \rightarrow s\gamma$ ), $\tan \beta = 50$



Scan over  $m_{1/2}, m_0, A_0$

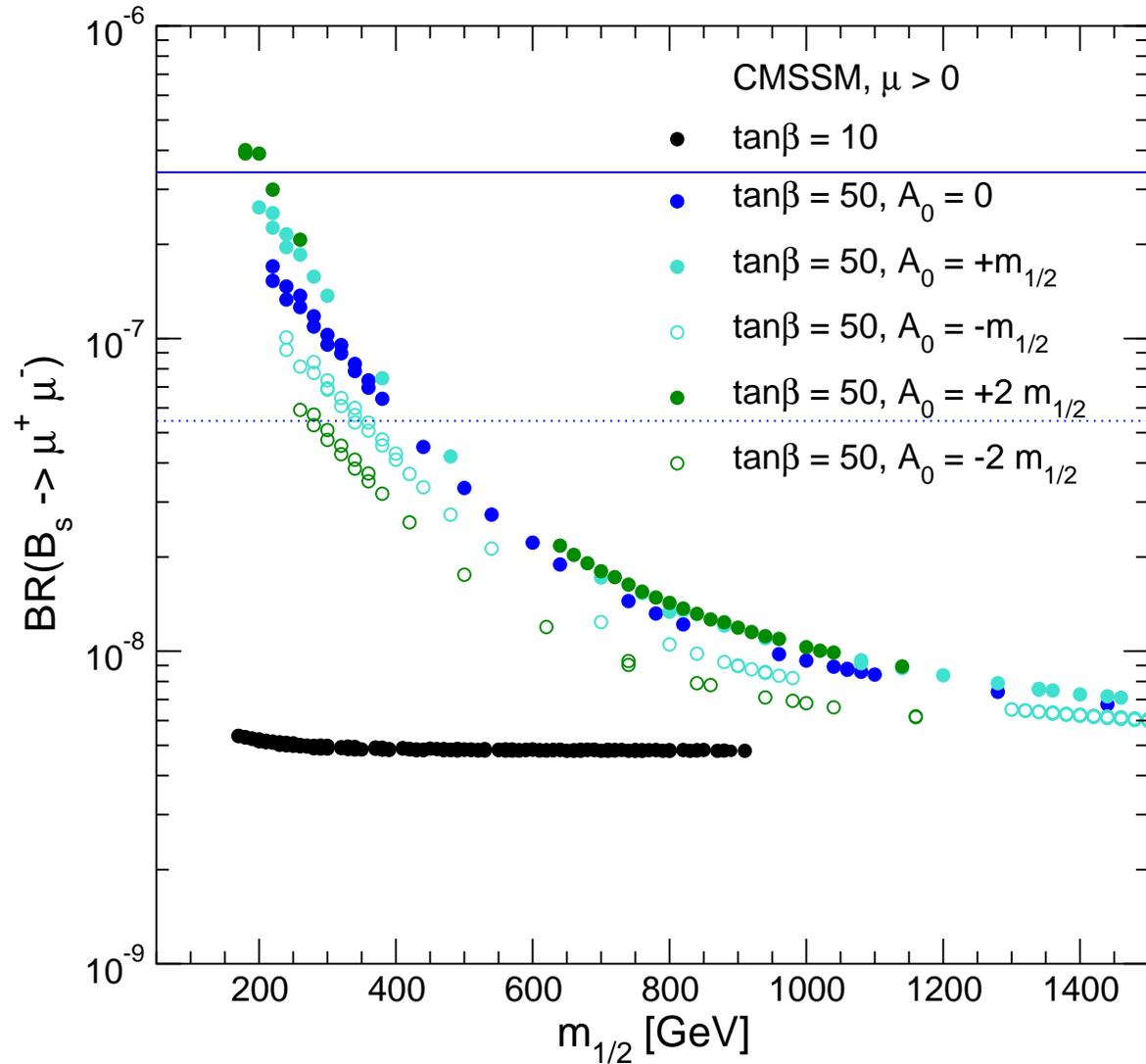
$\tan \beta = 50$

selected points give correct  
amount of cold dark matter

[Ellis, S.H., Olive, Weiglein '04]

# Investigation of mSUGRA with cold dark matter constraint:

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$$



Scan over  $m_{1/2}, m_0, A_0$

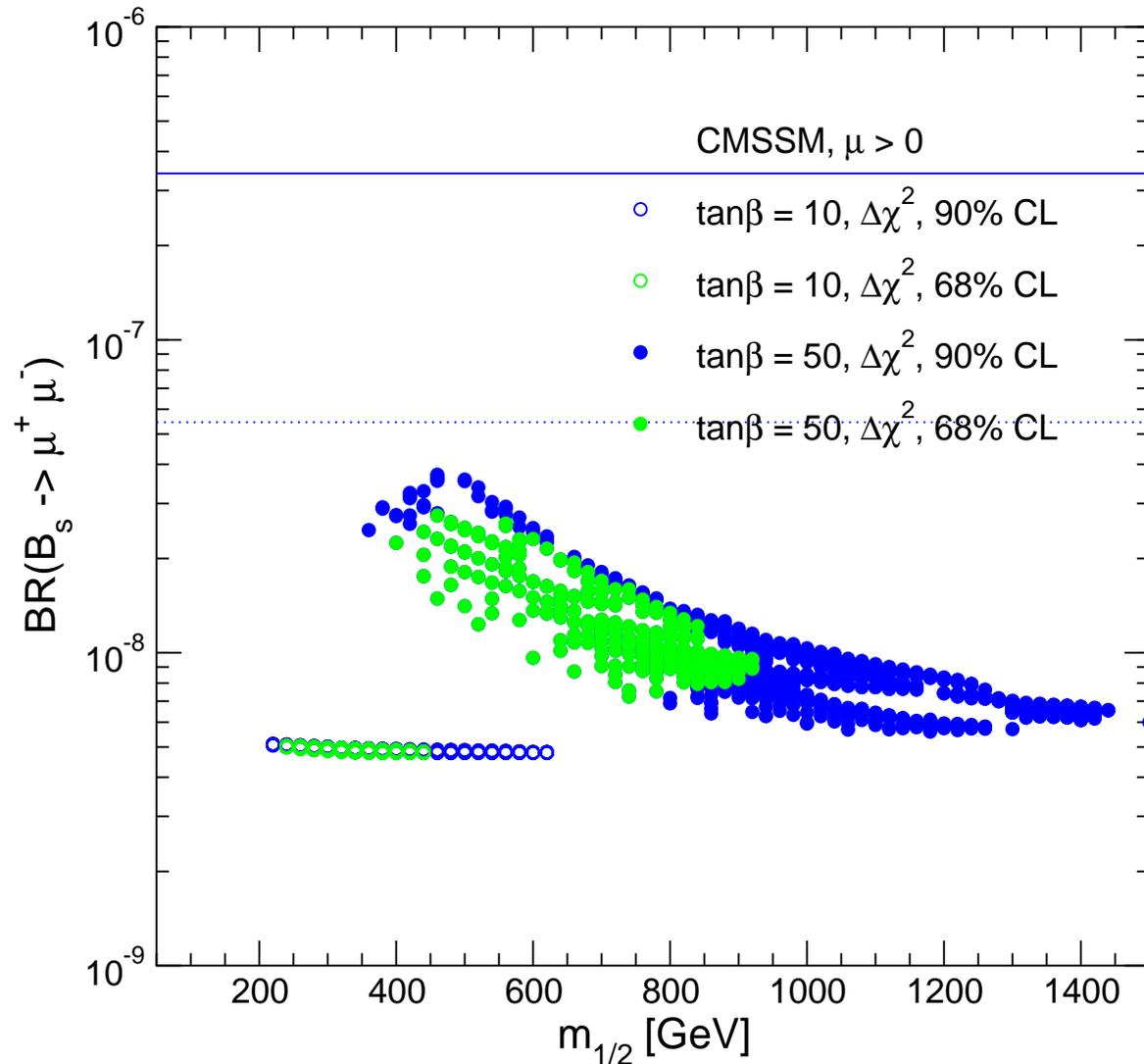
$\tan\beta = 10, 50$

selected points give correct amount of cold dark matter

[Ellis, S.H., Olive, Weiglein '04]

# Investigation of mSUGRA with cold dark matter constraint:

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$$



Scan over  $m_{1/2}, m_0, A_0$

$\tan\beta = 10, 50$

selected points give correct  
amount of cold dark matter

[Ellis, S.H., Olive, Weiglein '04]